

Because of the qualitative difference in the size of the mass and energy resolutions, we adopted a different procedure in correcting for them. In the case of the mass, the procedure used was one suggested by Terrell.⁵ Provided that over the range of the resolution function the observed distribution is everywhere adequately approximated by a polynomial of degree 3 or less, it can be shown for symmetrical resolution functions that the true distribution may be found from the observed G by the fold

$$g(x) = \int G(x-y)u(y)dy,$$

where $u(y)$ is an undispersing function. The only condition that must be satisfied by the function u is that it have a variance equal in size, but opposite in sign, to the resolution function.

The method outlined above is extremely neat and easy to use when it is applicable. However, it will lead to false results if the cubic expansion is insufficient, and this will often be the case when the resolution width is comparable to the intrinsic width. In fact, this was the

case of the energy resolution in our experiment. Of course, it is possible to allow for an expansion up to the fifth degree by including the fourth moments, but these are generally not well known. Although general methods are available involving Taylor's expansion, and consequently requiring the evaluation of the first and, perhaps, second derivatives of the experimental distribution function, the total kinetic energy distribution is sufficiently Gaussian that we may apply a useful but special method. If the resolution function is also Gaussian with a variance σ^2 , while the variance of the *observed* distributions is s^2 , then any linear function of the independent variable x given by $y = a + bx$ will be observed as a straight line with the equation

$$y_{\text{obs}} = a + bx(1 - \sigma^2/s^2),$$

where the origin has been taken at the position of the mean. In particular, the "calibration" equation $y = x$ will have its slope reduced to $(1 - \sigma^2/s^2)$. Thus, for Gaussian intrinsic distribution and resolution functions, any observed function of x may be resolution corrected simply by plotting the value at the corrected x point.

Lifetime Determination of the 2+ and 4+ Rotational Levels in Gd¹⁵⁴ and the Effect of Rotation-Vibration Interaction

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The lifetimes of the 2+ and 4+ rotational levels of Gd¹⁵⁴ were determined by coincidence measurements between the beta spectrum of Eu¹⁵⁴ and the conversion lines de-exciting the levels under investigation, utilizing a double-lens coincidence spectrometer. The measurements were carried out using the self-comparison method and a time-to-amplitude converter. The mean lifetimes of the 371- and the 123-keV levels were found to be $(5.6 \pm 0.7) \times 10^{-11}$ sec and $(1.67 \pm 0.07) \times 10^{-9}$ sec, respectively. The measured ratio for the reduced transition probabilities, $B(E2; 4+ \rightarrow 2+)/B(E2; 2+ \rightarrow 0+) = 1.77 \pm 0.25$, deviates from the value 1.43 predicted by the strict rotational model. The effect of rotation-vibration interaction on the transition probability ratios within a rotational band is discussed. These ratios are related uniquely to the distortion of the energy spacings. In the case of the ground-state band of Gd¹⁵⁴, theory thus predicts $B(E2; 4+ \rightarrow 2+)/B(E2; 2+ \rightarrow 0+) = 1.62$ in agreement with experiment.

INTRODUCTION

RECENTLY, with the improvement of lifetime measurement technique, some interest has been revived in lifetime determination of the two lowest excited states of even-even nuclei for strongly distorted nuclei. The strong-coupling model predicts a unique ratio between the reduced probabilities for transitions starting from the second 4+ excited state to the transitions from the first 2+ level. This theoretical ratio has been consistent within experimental error with values deduced from lifetime measurements in nuclei having a well-developed rotational spectra.¹⁻⁴

Very recent results for Dy¹⁶⁰, however, as discussed in the last section of this article, seem to indicate a discrepancy with theory.

In Os¹⁹⁰, where the energy interval rule is not strictly obeyed, it has been found¹ that the ratio of the transition probabilities is appreciably lower. This nucleus lies at the upper edge of a region of strongly distorted nuclei.

It was of great interest to find out this ratio in a nucleus lying at the lower side of this strong coupling region.

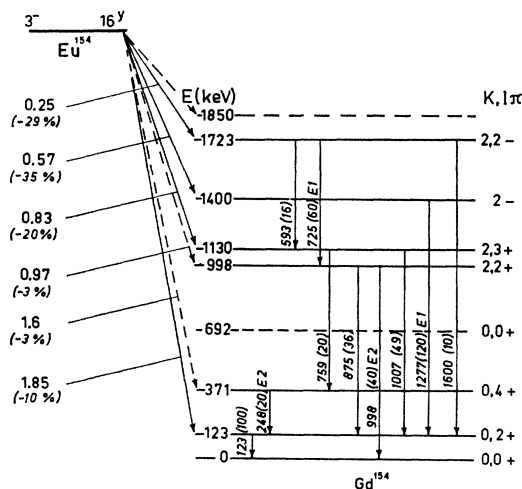
An abrupt transition in the properties of the nuclei occurs as the neutron number changes from 88 to 90. Sm¹⁵² and Gd¹⁵⁴, each of them having 90 neutrons, are at the verge of a region of strongly deformed nuclei. However, these nuclei displaying a considerable deviation from the rotational energy interval rule may still

¹ G. Scharff-Goldhaber, D. E. Alburger, G. Harbottle, and M. McKeown, Phys. Rev. **111**, 913 (1958).

² S. Ofer, Phys. Rev. **115**, 412 (1959).

³ J. Burde and M. Rakavy, Nucl. Phys. **28**, 172 (1961).

⁴ A. C. Li and A. Schwarzschild, Bull. Am. Phys. Soc. **7**, 359 (1962).

FIG. 1. Decay scheme of Eu^{154} .

be considered strongly deformed, but with strong rotation-vibration interaction.

In the present work we have measured the lifetime of the 371-keV ($4+$) excited state of Gd^{154} by the self-comparison method.⁵ We also remeasured the lifetime of the first excited ($2+$) state in the same experimental setup. Our result for the $2+$ state agrees very well with previous measurements.⁶⁻⁸ The ratio of the reduced transition probabilities

$$B(E2; 4+ \rightarrow 2+)/B(E2; 2+ \rightarrow 0+)$$

was found to be considerably higher than the value 1.43 predicted by the strict rotational model. Yet the experimental ratio seems to be consistent with the value calculated by taking into account the rotation-vibration interaction that explains the energy spectra. In the Appendix it is shown that under rather general assumptions the transition probability ratio may be related uniquely to the ratio of the energy-level spacings.

The energy levels of Gd^{154} populated by beta decay of Eu^{154} have been investigated previously.⁹⁻¹³ Figure 1 shows the decay scheme of Eu^{154} . A delayed coincidence measurement between beta particles and the conversion electrons of the 248-keV transition using the self-comparison method seemed to be the most appropriate for the lifetime measurement of the 371-keV $4+$ level,

⁵ R. E. Bell, R. L. Graham, and H. E. Petch, *Can. J. Phys.* **30**, 35 (1952).

⁶ A. W. Sunyar, *Phys. Rev.* **98**, 653 (1955).

⁷ E. E. Berlovich, *Izvest. Akad. Nauk S.S.S.R., Ser. Fiz.* **20**, 1438 (1956).

⁸ M. Birk, G. Goldring, and Y. Wolfson, *Phys. Rev.* **116**, 730 (1960).

⁹ J. M. Cork, M. K. Brice, R. G. Helmer, and D. E. Sarason, *Phys. Rev.* **107**, 1621 (1957).

¹⁰ J. O. Juliano and F. S. Stephens, *Phys. Rev.* **108**, 341 (1957).

¹¹ S. K. Bhattacharjee, Shree Raman, and S. K. Mitra, *Proc. Indian Acad. Sci.* **47**, 295 (1958).

¹² B. S. Dzelepov, N. N. Zhkovsky, V. G. Nedovesov, and G. E. Shukin, *Izvest. Akad. Nauk S.S.S.R., Ser. Fiz.* **21**, 966 (1957).

¹³ B. V. Bobikin and K. M. Novik, *Izvest. Akad. Nauk S.S.S.R., Ser. Fiz.* **21**, 1556 (1957).

provided that the lifetimes of the intermediate levels feeding it are sufficiently short in comparison.

Considering only those transitions which are coincident with β particles of about 200 keV (the energy of the continuous spectrum selected experimentally), we see that the 371-keV level is populated in about 96% of the total cases by direct beta transition and by the 758-keV transition from the 1129-keV level, which is itself fed by the 830-keV beta branch. The spin and parity assignment of the 998- and 1129-keV levels are $2+$ and $3+$, respectively, on the basis of directional correlation^{14,15} and conversion coefficient measurements.^{10,13} These levels are probably members of a gamma vibrational band with $K=2$. The $E2$ transitions de-exciting these levels to the ground-rotational band are expected to be enhanced by about a factor of ten in comparison with the single proton value.¹⁶ The 1129-keV level should have, therefore, a lifetime much shorter than the single particle value 6×10^{-12} sec.

In about 4% of the total cases the 371-keV level is fed, under the present experimental arrangement, by the 593 keV $E1$ transition from the 1723-keV level via the 1129-keV state. Baba and Bhattacharjee¹⁷ have assigned the spin and parity of the 1723-keV state as $2-$ on the basis of polarization correlation measurements and conversion coefficient data. This level has probably $K=2$ assignment and the $E1$ transitions de-exciting this state to the gamma vibrational band are not K forbidden. Owing to the relatively high energy of these $E1$ transitions, the lifetime of the 1723-keV state is expected to be very short.

Taking into account that only 4% of the 371-keV level is populated via the 1723-keV state, it is concluded that the uncertainty in the lifetime determination of the 371-keV level arising from the intermediate levels can be ignored in comparison with the experimental error.

EXPERIMENTAL ARRANGEMENT

The Eu^{154} source was prepared by irradiating a sample of Eu^{158} enriched to 97% in the form of oxide in the Saclay reactor for a period of two months at a flux of $2 \times 10^{18} \text{ n/cm}^2 \text{ sec}$.

The lifetimes of the two lowest excited states of Gd^{154} were determined by carrying out coincidence measurements between the beta spectrum and the conversion lines de-exciting the levels under investigation, using a double lens coincidence beta spectrometer.¹⁸ The transmission and momentum resolution in both spectrometers were adjusted to about 5%. A $20\text{-}\mu\text{C}$

¹⁴ G. D. Hickman and M. L. Wiedenbeck, *Phys. Rev.* **111**, 539 (1958).

¹⁵ P. Debrunner and W. Kundig, *Helv. Phys. Acta* **33**, 395 (1960).

¹⁶ A. Bohr and B. R. Mottelson, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 468.

¹⁷ C. V. K. Baba and S. K. Bhattacharjee, *Phys. Rev.* **123**, 865 (1961).

¹⁸ T. R. Gerholm, *Rev. Sci. Instr.* **26**, 1069 (1955).

source of Eu¹⁵⁴ in the form of chloride was deposited on a 1-mg/cm² Mylar foil covered with an evaporated thin film of aluminum. The source had a circular shape of about 5 mm in diameter.

The focused electrons were detected by plastic scintillators coupled to 56AVP Philips photomultipliers. The pulses from the collectors were fed into a delay to pulse-height converter similar to that described by Green and Bell.¹⁹ The pulses from the twelfth dynodes were introduced into linear amplifiers, and, after discrimination against the noise pulses, passed to a coincidence circuit having a resolving time of 10⁻⁷ sec. The outgoing pulses from the amplifiers were channeled within 30 to 40% width of the peak voltage. The pulses from the time-to-amplitude converter were fed into an 80-channel pulse-height analyzer and gated by the pulses from fast-slow coincidence circuit. A pile-up rejector circuit was also incorporated.

The performance of the experimental setup was tested outside the spectrometer with a Co⁶⁰ source, and the complete experimental arrangement was checked when a ThB source was inserted into the spectrometer, as described in detail previously.²⁰

In the self-comparison method,⁵ the position of the centroid of the analyzed spectrum, obtained by the gated multichannel analyzer at one setting of the spectrometer's current, was compared with the position of the centroid when the role of the spectrometers was inverted in time by an appropriate change in both channels. The shift between the centroid which corresponds to twice the mean lifetime of the excited state under investigation was calibrated by introducing a 7×10⁻¹⁰-sec delay cable between one of the limiters and the 6BN6 coincidence circuit. To eliminate time-dependent drifts, many sets of measurements of short duration were made. Every set included four measurements of 15 min each, and consisted of self-comparisons carried out for two different lengths of cables.

EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

The result obtained for the mean lifetime of the 371-keV level by analyzing a total number of 70 000 accumulated coincidences was $\tau = (5.6 \pm 0.7) \times 10^{-11}$ sec. In deriving these results, corrections were made for the fact that the spectrometer accepted about 7% of the total intensity of the conversion line when beta particles were focused at a position off the line, and also due to a change of 1×10^{-12} sec in the transit time of the electrons in the spectrometers at the two settings (before and after inversion). In estimating the error, the small uncertainty in determining the exact shape of the tail of the conversion line was also taken into account.

The lifetime measurement of the 123-keV 2+ level

was carried out by coincidence measurements between the beta spectrum and the *L* conversion line of the transition de-exciting the state. Analyzing a total number of 180 000 recorded coincidences the result obtained in the self-comparison method for the mean lifetime of the 123-keV level was $\tau = (1.65 \pm 0.1) \times 10^{-9}$ sec. The result was corrected for the fact that the spectrometer accepted about 11 and 4% of the total intensity of the conversion line at the lower and upper side of the line, respectively, at the positions off line. The change in the transit time of the electrons in the spectrometer at the two settings was less than 5×10^{-12} sec.

The lifetime of the first excited state was also determined from the slope of the decay curve recorded by the gated multichannel when one spectrometer was set on the *L* conversion line of the 123-keV transition and the other focused beta particles of about 160 keV. The result thus obtained for the mean lifetime is $\tau = (1.69 \pm 0.1) \times 10^{-9}$ sec. The combined result for the mean lifetime of the 123-keV state obtained by the two methods is $\tau = (1.67 \pm 0.07) \times 10^{-9}$ sec, which agrees fairly well with the results published previously.⁶⁻⁸

The theoretical values for the total conversion coefficients²¹ of the 123- and 248-keV transitions are 1.20 and 0.11, respectively; using these data and the results of the present measurements, the ratio of the reduced transition probabilities of the 248- to the 123-keV transition is given by

$$\frac{B(E2; 4+ \rightarrow 2+)}{B(E2; 2+ \rightarrow 0+)} = \frac{\tau_{123}(1 + \alpha_{123}^{\text{tot}})}{\tau_{248}(1 + \alpha_{248}^{\text{tot}})} \left(\frac{123}{248} \right)^5 = 1.77 \pm 0.25. \quad (1)$$

Recently Bernstein²² pointed out that there is a systematic deviation in the experimental conversion coefficients for rotational transitions from the corresponding theoretical values. In Gd¹⁵⁴ the experimental conversion coefficient for the 123-keV transition is higher by a factor of 1.2. Using this experimental value for the conversion coefficient the ratio of the reduced transition probabilities is increased by about 10%.

To compare with theory, it is convenient to define the ratios

$$\left[\frac{E(I)}{E_{\text{rot}}(I)} \right] / \left[\frac{E(I-2)}{E_{\text{rot}}(I-2)} \right] = 1 - \epsilon(I), \quad (2)$$

$$\left[\frac{B(E2; I \rightarrow I-2)}{B_{\text{rot}}(E2; I \rightarrow I-2)} \right] / \left[\frac{B(E2; I-2 \rightarrow I-4)}{B_{\text{rot}}(E2; I-2 \rightarrow I-4)} \right] = 1 + \eta(I),$$

¹⁹ R. E. Green and R. E. Bell, Nucl. Inst. 3, 127 (1958).

²⁰ J. Burde, M. Rakavy, and G. Engler, Phys. Rev. 128, 325 (1962).

²¹ L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Report, 1956 [translation: Report 57 ICCK 1 and ICCL 1, issued by Physical Department, University of Illinois, Urbana, Illinois (unpublished)].

²² E. M. Bernstein, Phys. Rev. Letters 8, 100 (1962).

where $E(I)$ is the actual excitation energy of the rotational level of spin I and $E_{\text{rot}}(I)$ is the corresponding value as given by the strict rotational model. $B(E2; I \rightarrow I-2)$ and $B_{\text{rot}}(E2; I \rightarrow I-2)$ are the actual and strict rotational model values of the reduced transition probabilities from level with spin I to that of spin $(I-2)$.

A calculation of the rotation-vibration interaction, given in the Appendix, yields the following relation between $\eta(I)$ and $\epsilon(I)$:

$$\eta(I)/\epsilon(I) = 2(2I-3)/(2I-1). \quad (3)$$

In deriving this relation it is necessary to assume only the specific functional form of the dependence of the components of the moment of inertia on the deformation parameters β and γ as given by the hydrodynamical model. Under this specific assumption the effects of the β and γ vibrations enter in the same combination into the expressions for $\eta(I)$ and $\epsilon(I)$. Hence, the ratio $\eta(I)/\epsilon(I)$ is independent of any specific assumptions on the vibrational part of the Hamiltonian and, consequently, does not depend on the structure of the perturbing vibrational states.

Using the rather exact experimental value $\epsilon=0.095$, theory predicts $\eta=0.135$ or, equivalently,

$$\left[\frac{B(E2; 4+ \rightarrow 2+)}{B(E2; 2+ \rightarrow 0+)} \right]_{\text{theor}} = 1.62. \quad (4)$$

This ratio agrees with the experimental value given above within the accuracy of the measurements.

In Dy¹⁶⁰ an agreement has been found with the theoretical ratio within experimental error when the lifetime of the 2+ state was taken from earlier measurements.³ But if this value is deduced from a recent lifetime measurement of the first excited state,²³ a deviation of about 20% is noticed when the theoretical value is taken for the conversion coefficient. However, this ratio is strongly dependent on the conversion coefficient of the lower excited state which is very high in the present case, and the result would appear as a clear discrepancy only if a direct and precise measurement of the conversion coefficient would confirm the theoretical value.²¹ The deviation from theory for this nucleus which has well-developed rotational spectrum is rather surprising and clearly cannot be explained by the rotation-vibration interaction presented here.

APPENDIX

The distortion of the energy spacings in the ground-state rotational band of an even nucleus is proportional to $[I(I+1)]^2$ with a coefficient which is the sum of two terms, originating one from the interaction of the rotation with β vibrations and the other from the interaction with γ vibrations.²⁴ The deviations of the $E2$

transition probabilities within this band from the pure rotational values can be expressed by the same parameters and could in principle be used to separate the effects of the β and γ vibrations. Yet, as we show below, the ratios between the transition probabilities depend on the same combination of the parameters and can be expressed uniquely by the ratio of the energies.

In order to derive the distortions in the energy levels and the transition probabilities we have to assume a *specific functional dependence* of the components of the moment of inertia on the coordinates β and γ .²⁵ This functional dependence we take from the hydrodynamical model.²⁴

$$\begin{aligned} g_1(\beta_0, \gamma=0) &= g_2(\beta_0, \gamma=0) \equiv g_0, \\ \left[\frac{\partial}{\partial \beta} g_1 \right]_{\beta_0, \gamma=0} &= \left[\frac{\partial}{\partial \beta} g_2 \right]_{\beta_0, \gamma=0} = 2g_0, \\ \left[\frac{\partial}{\partial \gamma} g_1 \right]_{\beta_0, \gamma=0} &= - \left[\frac{\partial}{\partial \gamma} g_2 \right]_{\beta_0, \gamma=0} = \frac{2}{\sqrt{3}}g_0. \end{aligned} \quad (A1)$$

The Hamiltonian of the collective coordinates may be written as follows:

$$H = H_0 + H_1, \quad (A2)$$

$$\begin{aligned} H_0 &= T_{\text{vib}} + \frac{1}{2}\hbar^2 I_3^2 \left(\frac{1}{g_3(\beta, \gamma)} - \frac{1}{g_0} \right) + V(\beta, \gamma) + \frac{\hbar^2}{2g_0} I^2 \\ &= H(\beta, \gamma) + \frac{\hbar^2}{2g_0} I^2, \end{aligned} \quad (A3)$$

$$\begin{aligned} H_1 &= \left\{ (\beta - \beta_0) \frac{\partial}{\partial \beta} + \gamma \frac{\partial}{\partial \gamma} \right\} \frac{1}{2} \hbar^2 \left(\frac{I_1^2}{g_1} + \frac{I_2^2}{g_2} \right) \Big|_{\beta=\beta_0, \gamma=0} \\ &= -2 \frac{\hbar^2 (I^2 - I_3^2)}{2g_0} \frac{\beta - \beta_0}{\beta_0} - \frac{2}{\sqrt{3}} \frac{\hbar^2 (I_1^2 - I_2^2)}{2g_0} \gamma. \end{aligned} \quad (A4)$$

The explicit form of $H(\beta, \gamma)$ is of no concern to us, but one should notice that it depends on the eigenvalue K of I_3 .

The eigenfunctions of $H(\beta, \gamma)$ are designated by K and some other quantum numbers α :

$$H(\beta, \gamma) \varphi_{\alpha K} = E_{\alpha K} \varphi_{\alpha K}. \quad (A5)$$

For the ground state we define $\alpha=0$, $E_{00}=0$. The eigenvalues and eigenfunctions of H_0 are defined by

$$\begin{aligned} H_0 \phi(\alpha, K, I, M) &= [E_{\alpha K} + E_{\text{rot}}(I)] \phi(\alpha, K, I, M), \\ E_{\text{rot}}(I) &= (\hbar^2/2g_0) I(I+1), \end{aligned} \quad (A6)$$

and the matrix elements of the perturbing Hamiltonian H_1 between the ground-state band and other bands may be expressed as follows:

²³ D. B. Fossan and B. Herskind, Phys. Letters 2, 155 (1962).

²⁴ A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, No. 16 (1953).

²⁵ We follow the notation of reference 24.

$$\begin{aligned} &\langle \phi(\alpha, K, I, M) | H_1 | \phi(0, 0, I, M) \rangle \\ &= -2E_{\text{rot}}(I) \left\langle \alpha K \left| \frac{\beta - \beta_0}{\beta_0} \right| 00 \right\rangle \delta_{K,0} \\ &-\frac{\sqrt{2}}{\sqrt{3}} E_{\text{rot}}(I) \left(1 - \frac{2}{I(I+1)} \right)^{1/2} \langle \alpha K | \gamma | 00 \rangle \delta_{K,2}. \end{aligned} \quad (\text{A7})$$

The perturbation in the energy to lowest order is²⁶

$$\begin{aligned} \Delta E = -[E_{\text{rot}}(I)]^2 &\left\{ 4 \sum_{\alpha} \frac{|\langle \alpha 0 | (\beta - \beta_0) / \beta_0 | 00 \rangle|^2}{E_{\alpha 0}} \right. \\ &\left. + \frac{2}{3} \left[1 - \frac{2}{I(I+1)} \right] \sum_{\alpha} \frac{|\langle \alpha 2 | \gamma | 00 \rangle|^2}{E_{\alpha 2}} \right\}. \end{aligned} \quad (\text{A8})$$

Defining the parameters

$$b = \sum_{\alpha} \frac{|\langle \alpha 0 | (\beta - \beta_0) / \beta_0 | 0, 0 \rangle|^2}{(E_{\alpha 0} / E_{\text{rot}}(I = 2))}, \quad c = \sum_{\alpha} \frac{|\langle \alpha 2 | \gamma | 0, 0 \rangle|^2}{(E_{\alpha 2} / E_{\text{rot}}(I = 2))},$$

and leaving out terms proportional to E_{rot} , the ratio of the perturbed energies to the pure rotational ones is given by

$$E(I) / E_{\text{rot}}(I) = 1 - (2/3)I(I+1)[b + (1/6)c]. \quad (\text{A10})$$

The reduced $E2$ transition probabilities are proportional to the square of the reduced matrix elements of the quadrupole operator

$$\begin{aligned} M_{\mu} &= \frac{3ZeR^2}{4\pi} \beta \left[\cos \gamma D_{\mu 0^2} + \frac{1}{\sqrt{2}} \sin \gamma (D_{\mu 2^2} + D_{\mu -2^2}) \right] \\ &\cong \frac{3ZeR^2}{4\pi} \beta_0 \left[\left(1 + \frac{\beta - \beta_0}{\beta_0} \right) D_{\mu 0^2} + \frac{\gamma}{\sqrt{2}} (D_{\mu 2^2} + D_{\mu -2^2}) \right]. \end{aligned} \quad (\text{A11})$$

The perturbed eigenfunctions are, to first approximation, $\psi(0, 0, I, M) = \phi(0, 0, I, M)$

$$\begin{aligned} &+ 2E_{\text{rot}}(I) \sum_{\alpha} \frac{\langle \alpha 0 | (\beta - \beta_0) / \beta_0 | 00 \rangle}{E_{\alpha 0}} \phi(\alpha, 0, I, M) \\ &+ \frac{\sqrt{2}}{\sqrt{3}} E_{\text{rot}}(I) \left[1 - \frac{2}{I(I+1)} \right]^{1/2} \\ &\times \sum_{\alpha} \frac{\langle \alpha 2 | \gamma | 00 \rangle}{E_{\alpha 2}} \phi(\alpha, 2, I, M). \end{aligned} \quad (\text{A12})$$

²⁶ Terms proportional to $E_{\text{rot}}(I)$ arising from the first-order perturbation caused by higher order terms of the expansion of $\mathcal{H}_K(\beta, \gamma)$ in powers of $(\beta - \beta_0)$ and γ are left out.

Hence we get

$$\begin{aligned} &\left[\frac{B(E2; I_i \rightarrow I_f)}{B_{\text{rot}}(E2; I_i \rightarrow I_f)} \right]^{1/2} \\ &= \frac{(\psi_i || M || \psi_f)}{(3/4\pi)ZeR^2\beta_0(2I_f+1)^{1/2}(20I_f0|I_i,0)} \\ &= 1 + \frac{1}{3}[I_i(I_i+1) + I_f(I_f+1)]b \\ &+ \frac{1}{6} \frac{\sqrt{2}}{\sqrt{3}} \left\{ \frac{(22I_f0|I_i,2)}{(20I_f0|I_i,0)} I_i(I_i+1) \left[1 - \frac{2}{I_i(I_i+1)} \right]^{1/2} \right. \\ &\left. + \frac{(22I_f-2|I_i,0)}{(20I_f0|I_i,0)} I_f(I_f+1) \left[1 - \frac{2}{I_f(I_f+1)} \right]^{1/2} \right\} c. \end{aligned} \quad (\text{A13})$$

$B(E2; I_i \rightarrow I_f)$ and $B_{\text{rot}}(E2; I_i \rightarrow I_f)$ are the perturbed and unperturbed transition probabilities.

Inserting the explicit expressions for the Clebsch-Gordan coefficients, one obtains to first-order accuracy

$$\begin{aligned} &B(E2; I \rightarrow I-2) / [B_{\text{rot}}(I \rightarrow I-2)] \\ &= 1 + \frac{4}{3}I(I-1)[b + \frac{1}{6}c] + \frac{4}{3}[b + \frac{2}{3}c]. \end{aligned} \quad (\text{A14})$$

To the same accuracy the ratios $\epsilon(I)$ and $\eta(I)$ defined by Eq. (2) depend only on $(b + \frac{1}{6}c)$:

$$\begin{aligned} \epsilon(I) &= \frac{4}{3}(2I-1)[b + \frac{1}{6}c], \\ \eta(I) &= \frac{8}{3}(2I-3)[b + \frac{1}{6}c], \end{aligned} \quad (\text{A15})$$

or

$$\eta(I) / \epsilon(I) = 2(2I-3) / (2I-1). \quad (\text{A16})$$

Assuming for $V(\beta, \gamma)$ a quadratic form and making use of the explicit form²⁷ of $H(\beta, \gamma)$ in Eq. (A3), the parameters b and c defined in Eq. (A9) may be related to the rotational and vibrational energies

$$\begin{aligned} b &= \frac{1}{2}[E_{\text{rot}}(I=2) / \hbar\omega_{\beta}]^2, \\ c &= [E_{\text{rot}}(I=2) / \hbar\omega_{\gamma}]^2. \end{aligned} \quad (\text{A17})$$

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²⁷ A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 26, No. 14 (1952).